2.3 A brief summary of Stern-Gerlach experiments

- The physical picture of the experiment gets translated mathematically to the set of vectors: $\mathbf{S_z^+}$, $\mathbf{S_z^-}$, $\mathbf{S_x^+}$, $\mathbf{S_x^-}$, etc. [Later we will represent these vectors using the odd-notation: $|\mathbf{S_z^+}\rangle$, $|\mathbf{S_z^-}\rangle$, $|\mathbf{S_x^+}\rangle$, $|\mathbf{S_x^-}\rangle$, etc. But for now let us ignore this detail.)
- The space represented by these vectors is *an abstract two-dimensional space*. Note: the axes in this abstract space: $\mathbf{S_z^+}$, $\mathbf{S_z^-}$, for example, are not the same axes where the magnetic fields are aligned.
- This abstract space is a portion of what we will later begin to describe as the Hilbert space.
- It appears that the "projection" process we discussed in this space completely describes what happens in the measurements involved during Stern-Gerlach experiments.
- But, this space is complex. Hence, it is difficult to properly draw it on a board (because it is two-dimensional *and* complex). However, we may be able to visualize it in our heads.
- Certain words are now limiting because of the presence of complex numbers. The words, angle, or dot product were originally developed for real-space vectors and we will need to generalize these definitions.
- We find that the following represents a mathematically consistent description of what is happening in these experiments:

  Stern-Gerlach spin states $\rightarrow$ vectors (in a complex space).
  Measurement $\rightarrow$ projection
  Note also that the projection appears to occur onto special directions!!
  Projected components $\rightarrow$ complex numbers in general
  Measured quantities $\rightarrow$ absolute values of projected components.

  Hence, although the projected components may be complex, the measured values are *generally* real.
- While we have based our study thus far on the Stern Gerlach experiments, which deal with spin, the above description is appropriate for *all kinds of measurable quantities*.
  - We will late introduce an arbitrary vector that describes the electronic properties of a system and even that vector resides in the complex Hilbert space we described above.
  - An experiment conducted on that vector is a projection similar to what we have introduced above.
- The above ideas will lead us to "the postulates of quantum mechanics".

