C Complex variables

1. Imaginary numbers are depicted using the measure: $\imath = \sqrt{-1}$. Hence a complex number is one that has real and imaginary parts: $z = x + \imath y$. A complex number can be represented on a two-dimensional graph, with the vertical axis being called the imaginary axis and the horizontal axis called the real axis.

2. **Homework: Draw a two-dimensional plot and represent the two complex numbers $z_1 = x_1 + \imath y_1$ and $z_2 = x_2 + \imath y_2$ on the plot. For arbitrary values.**

3. The magnitude of a complex number, z, is simply the radius of the circle that contains z. Thus $|z| = x^2 + y^2$. The quantity $|z|$ is the magnitude of the complex number z.

4. The sum of two complex numbers is simply the sum of the respective real and imaginary parts. That is $z = z_1 + z_2 = (x_1 + x_2) + \imath(y_1 + y_2)$.

5. Hence complex numbers in a sense do look like vectors.

6. The complex conjugate of the complex number z is denoted as z^* and is defined as $z^* = x - \imath y$. Notice that the sign of the imaginary part is flipped during the process of complex conjugation. .

7. **Homework: Product of complex numbers: Let, $z_1 = x_1 + \imath y_1$ and $z_2 = x_2 + \imath y_2$. Multiply z_1 and z_2. Be careful.**

8. **Homework: Show that $|z| = z^* z = x^2 + y^2$.**

9. Let r be the radius of the circle: $r = |z| = x^2 + y^2$. Let θ be the angle between the line connecting z to the origin and the horizontal axis. **Homework: Show that $z = r\cos\theta + \imath r\sin\theta$.** This homework leads to the Euler's identity: $\cos\theta + \imath\sin\theta = \exp\imath\theta$. Hence; $z = r\exp\imath\theta$. This is called the polar representation of complex numbers.

10. **Homework: Using only the polar representation, show that $|z| = r$.**

11. **Homework: Use the polar representation and show that $\imath^{1/2} = \frac{1}{\sqrt{2}}(1 + \imath)$.**

